KARNATAKA ICSE SCHOOLS ASSOCIATION

ISC STD. XII   Preparatory Examination 2024

Subject – Mathematics

Time Allowed: 3 hrs **Maximum Marks: 80** **Date: 10 .01.2024**

(*Candidates are allowed* ***additional 15 minutes*** *for* ***only*** *reading the paper.*
They must ***NOT*** *start writing during this time*.)
This Question Paper consists of three sections A, B and C
Candidates are required to attempt all questions from ***Section A*** *and all questions*
EITHER *from* ***Section B OR Section C****.*
Section A*: Internal choice has been provided in* ***two questions of two marks each, two questions of four marks each and two questions of six marks each.***
Section B*: Internal choice has been provided in* ***one question of two marks and one question of four marks.***
Section C*: Internal choice has been provided in* ***one question of two marks and one question of four marks.***
All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.
The intended marks for questions or parts of questions are given in brackets [ ].
Mathematical tables and graph papers are provided.

SECTION A (65 marks)

Question 1

In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed. $[1 \times 15 = 15]$

(i) If $f: R \to R$ is invertible and is defined as $f(x) = \sqrt{x+2}$, then $f^{-1}(x)$ is :

(a) $(x+2)^2$
(b) $\sqrt{x+2}$
(c) $x^2 + 2$
(d) $x^2 - 2$

(ii) The value of $sin^{-1}\left(\frac{1}{3}\right) + sec^{-1}(3)$ is :

(a) $\frac{\pi}{3}$

(b) 0

(c) $\frac{\pi}{2}$

(d) $\frac{\pi}{6}$

(iii) A differentiable function is:

(a) Always continuous.

(b) Sometimes continuous.

(c) Never continuous.

(d) None of the above.

(iv) If matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, then the value of A^3 is:

(a) $\begin{pmatrix} 1 & 0 \\ 8 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix}$

(d) $\begin{pmatrix} 6 & 1 \\ 0 & 1 \end{pmatrix}$

(v) The function $f(x) = x^3 - 9x$ is decreasing in the interval:

(a) $\left(-\sqrt{2}, \sqrt{3}\right)$

(b) $(0,\ 3)$

(c) $\left(-\sqrt{3}, 3\right)$

(d) $(-\sqrt{3}, \sqrt{3})$

(vi) If $f(x) = x^2 + 7$ and $g(x) = |x|$ then the value of $f(3) \times g(-4)$ is:

(a) 24
(b) 64
(c) 4
(d) -64

(vii) The derivative of $y = \log x - \frac{1}{x}$ is :

(a) $\frac{1}{x} - \frac{1}{x^2}$

(b) $x - \frac{1}{x^2}$

(c) $x + \frac{1}{x^2}$

(d) $\frac{1}{x} + \frac{1}{x^2}$

(viii) Assertion: The function $f(x) = \frac{1}{x-1}$ is not continuous $\forall\ x \in R$

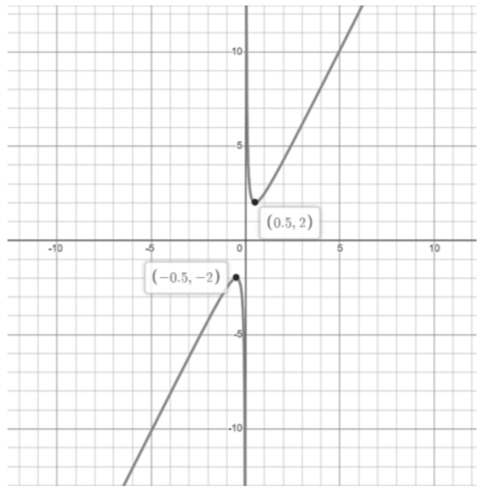
Reason: $f(x)$ is not differentiable at $x = 1$.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

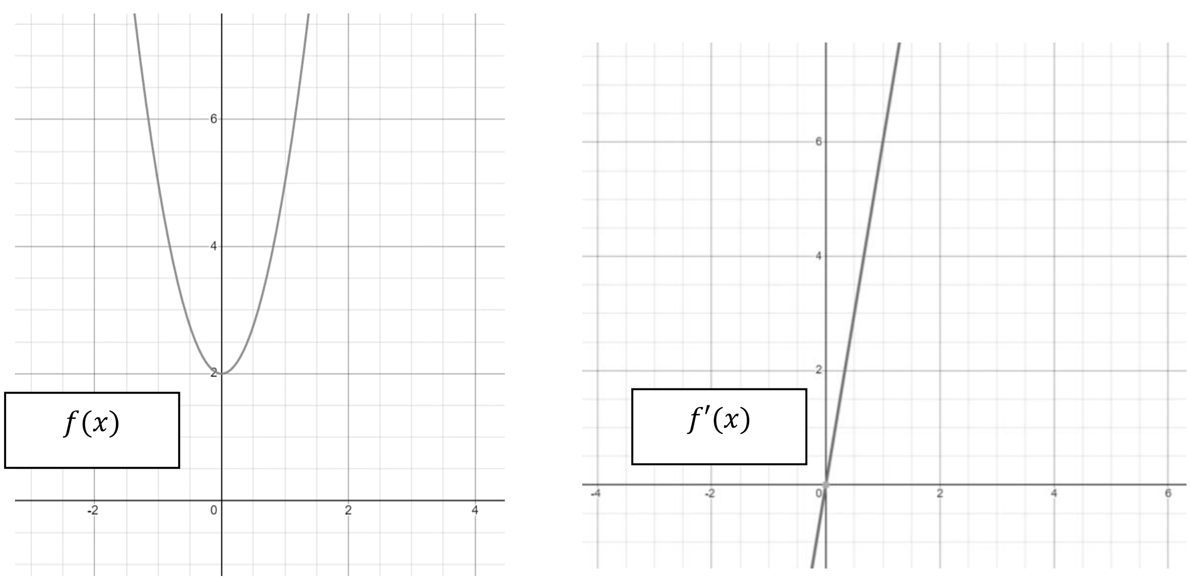
(ix) The degree of the differential equation $\sqrt{\frac{d^2y}{dx^2}} + \left(\frac{dy}{dx}\right)^4 + 16x = 5y$ is :
(a) 1
(b) 3
(c) 2
(d) 0.5

(x) For a function having the graph below, the local maximum value is:



(a) -0.5
(b) -2
(c) 0.5
(d) 2

(xi) The graph of a function $f(x)$ and that of its derivative $f'(x)$ are shown below. State if the association is correct or not.



(xii) If $\begin{pmatrix} 1 & 2 \\ 5 & 6 \end{pmatrix} = X + Y$ where X is a symmetric matrix and Y is a skew-symmetric matrix, find the value of $|X|$.

(xiii) If $\int x.e^x dx = x.e^x + k + c$ where k is a function of x, find the value of k.

(xiv) Four cards are drawn successively one after another from a well- shuffled pack of 52 cards. If the cards are not replaced, find the probability that all of them are aces.

(xv) Sapnil speaks the truth in 30% of the cases and Vihaan speaks the truth in 60% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?

Question 2 [2]

(i) If $x = e^{x-2y}$, then find $\frac{dy}{dx}$.

OR

(ii) Find the coordinates of the point on the curve $y = x^2 - 2$ where the rate of change of the ordinate is twice that of the rate of change of abscissa.

Question 3 [2]

Using properties of determinants, show that $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$

Question 4 [2]

Let $f: R \to R$ be defined as $f(x) = \frac{5}{x-2}$, find: the domain and range of $f^{-1}(x)$.

Question 5 [2]

Solve the differential equation $2\frac{dy}{dx} - x = e^x$

Question 6 [2]

(i) Find the value of $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx$

OR

(ii) Evaluate $\int_{-4}^{4} |x+3|\, dx$

Question7 [4]

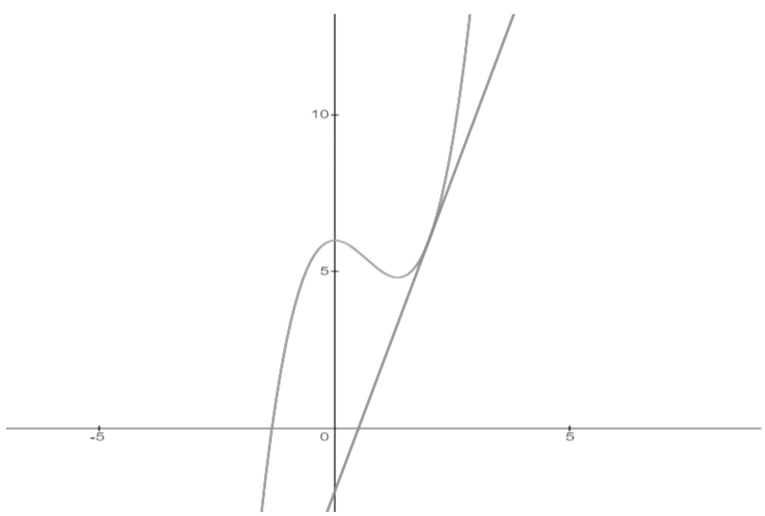
Solve the differential equation $\frac{dy}{dx} = \frac{3x^2-y^2}{2xy}$

Question 8 [4]

Solve for x, if $\tan^{-1}\left(\frac{x-2}{x-4}\right) + \tan^{-1}\left(\frac{x+2}{x+4}\right) = \frac{\pi}{4}$.

Question 9 [4]

(i) Find the equations of the tangents to the curve $y = x^3 - 2x^2 + 6$ which are parallel to the line $4x - y + 4 = 0$



OR

(ii) A stone dropped into still water produces a series of continually enlarging concentric circles; it is required to find the rate per second at which the area of one of them is enlarging when its diameter is 14 cm, supposing the wave to be then receding from the centre at the rate of 4 cm/sec.

Question 10 [4]

(i) A coin is tossed and if the coin shows head it is tossed again but if it shows a tail then a die is tossed. If 8 possible outcomes are equally likely, find the probability that the die shows a number greater than 4 if it is known that the first throw of the coin results in a tail.

OR

(ii) It is known that 60% students in a college reside in hostel while remaining 40% are day scholars. 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade. If one randomly chosen student has A grade, what is the probability that he lives in the hostel?

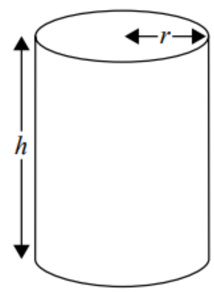
Question 11 [6]

Solve the system of equations using the matrix method

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4, \qquad \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1, \qquad \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

Question 12 [6]

(i) A closed cylindrical can with radius r centimetres and height h centimetres has a volume of $20\pi\ cm^2$.



(a) Express h in terms of r.

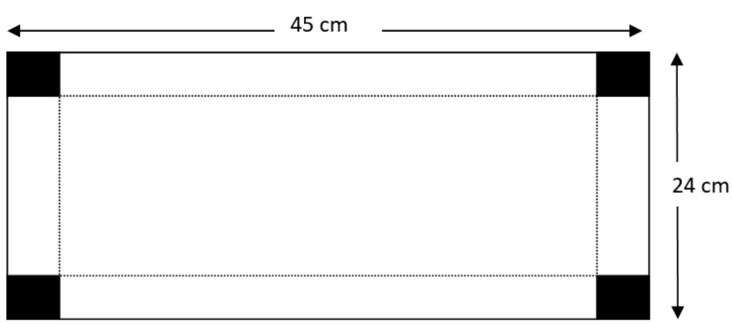
The material for the base and top of the can costs ₹ 2 per cm^2 and the material for the curved side costs ₹ 5 per cm^2.

(b) Find the total cost C as a function of r.

(c) Given that there is a minimum value of C. Find the minimum value in terms of π

OR

(ii) A rectangular sheet of tin 45 cm by 24 cm is to be made into boxes without top, by cutting off squares from the corners and folding the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum possible? Also find the maximum volume.



Question 13 [6]

(i) Find the particular solution of the differential equation $(x+1)\frac{dy}{dx} = 2xy$, given that $y(2) = 3$.

OR

(ii) Find the integration $\int \frac{x^2}{(x^2-2)(x^2+4)} dx$

Question 14 [6]

From a list containing 30 items, 4 of which are defective, 4 are chosen at random. Let X be the number of defectives found. Find the probability distribution of X , if the items are chosen without replacement. Also find the mean of the probability distribution.

SECTION B (15 marks)

Question 15 [5]

In sub parts (i) and (ii) choose the correct answer and answer the other subparts as instructed.

(i) Given that $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 4\,\hat{k}$,$\vec{b} = \hat{\imath} + \hat{\jmath} + 4\,\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath}$.
If $\vec{a} - t\vec{c}$ is perpendicular to $\vec{\boldsymbol{b}}$ then the value of $\boldsymbol{t}$ is

(a) 11
(b) 15
(c) 19
(d) 21

(ii) The planes $2x - 3y + 9z = 8$ and $3x - 4y - 2z = 9$ are:

(a) Parallel to each other.
(b) Perpendicular to each other.
(c) Having $x -$ axis as the line of intersection.
(d) Having $x -$ axis as the line of intersection.

(iii) Find a vector of magnitude 5 units parallel to the vector $2\hat{\imath} - \hat{\jmath} + 4\hat{k}$.

(iv) Find the equation of the plane with intercept 5 on the $y -$ axis and parallel to XZ plane.

(v) Find the position vector of the point R which divides the join of $\overrightarrow{OP} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $\overrightarrow{OQ} = 2\hat{\imath} + \hat{k}$ internally in the ratio $1:2$.

Question 16 [2]

(i) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then show that $(\vec{r} \times \hat{i})^2 = y^2 + z^2$

OR

(ii) Find the unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ where $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} - 2\hat{k}$.

Question 17 [4]

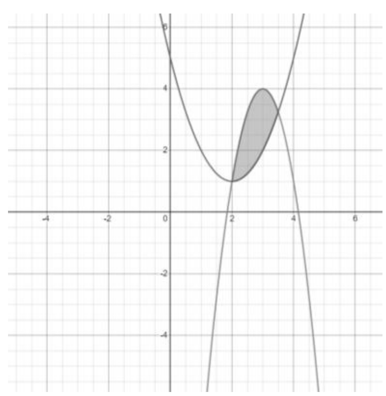
(i) Find the shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$.

OR

(ii) Find the Cartesian equation of the plane passing through the intersection of the planes $\vec{r}.(2\hat{i} + 6\hat{j}) + 12 = 0$ and $3x - y + 4z = 0$ and passing through the point $(1,\ 1,1)$.

Question 18 [4]

Find the area enclosed by the curves $y = -3(x-3)^2 + 4$ and $y = (x-2)^2 + 1$ as shown in the shaded region.



SECTION C (15 marks)

Question 19 [5]

In subparts (i) and (ii) choose the correct answer and answer the other subparts as instructed.

(i) If the coefficient of correlation $r = 1$ then the two lines of regression are:

(a) Parallel to $x -$ axis.

(b) Parallel to $y -$ axis.

(c) Parallel to each other.

(d) Coincident.

(ii) For break-even points,

(a)$R(x) > C(x)$.

(b)$R(x) < C(x)$.

(c)$R(x) = C(x)$.

(d)$R(x) = \frac{1}{2}C(x)$.

(iii) The total revenue of a commodity is given by $R(x) = ax - 2b$. Find the demand function when the marginal revenue is zero.

(iv) For the two lines of regression $x - 11y = 13$, $2x - 3y = 11$, find $\boldsymbol{r}$, the coefficient of correlation.

(v) If the total cost function for a manufacturer is given by $C(x) = \frac{x+1}{x}$, find the marginal cost function.

Question 20 [2]

(i) A car battery manufacturer finds that the total cost of producing and marketing x batteries is $C(x) = 500x^2 + 1500x - 3000$. Each product is sold for ₹4,000.

Determine the break-even points.

OR

(ii) If the total cost function is given by $C = a + bx + cx^2$, where x is the quantity of output, show that $\frac{d}{dx}(AC) = \frac{1}{x}(MC - AC)$, where MC is the marginal cost and AC is the average cost.

Question 21 [4]

If two lines of regression are $x - 3y - 5 = 0$ and $2x - 3y - 8 = 0$ and the variance of x is 5,

Find the variance of y and the coefficient of correlation.

Question 22 [4]

(i) A dealer wishes to purchase a number of fans and sewing machines. He has only ₹57,600 to invest and has space for at most 20 items. A fan costs him ₹3600 and a sewing machine ₹2400. He expects to sell a fan at a profit of ₹220 and a sewing machine at a profit of ₹180. Assuming that he can sell all the items that he buys, how should he invest his money to maximize the profit? Solve graphically and find the maximum profit.

OR

(ii) A man has ₹1500 for purchase of rice and wheat. A bag of rice and a bag of wheat cost ₹180 and ₹ 120 respectively. He has storage capacity of 10 bags only. He earns a profit of ₹ 11 and ₹9 per bag of rice and wheat respectively. Formulate an L.P.P to maximize the profit and solve it.